Investigation of Asymmetry in Particle Collisions

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Abstract

The forward-backward asymmetry for the process $e^+e^- \rightarrow \mu^+\mu^-$ is investigated by simulating a particle collider and a detector running in the centre-of-mass energy range $20 - 140\ GeV$. The asymmetry ratio is extracted from a curve fit to the angular distribution of detected muon pairs for different energies and detector resolutions. Then, the "r" and "j" parameters of the scattering matrix are obtained from a fit to the simulated data of asymmetry ratio with corresponding centre-of-mass energies. The computational results of these parameters show a good agreement with the values obtained from CERN in 1997.

Introduction

When an electron and a positron are collided at high energies, they annihilate to form a quantum superposition of a Z boson and a photon [1]. The composition of this superposition depends on the centre-of-mass energy of the collision. Subsequently, the quantum superposition decays into a pair of muons provided with enough energy [1]. The Feyman diagram for this process is shown in Fig. 1. A[1], in which horizontal axis represents time and vertical axis represents space.

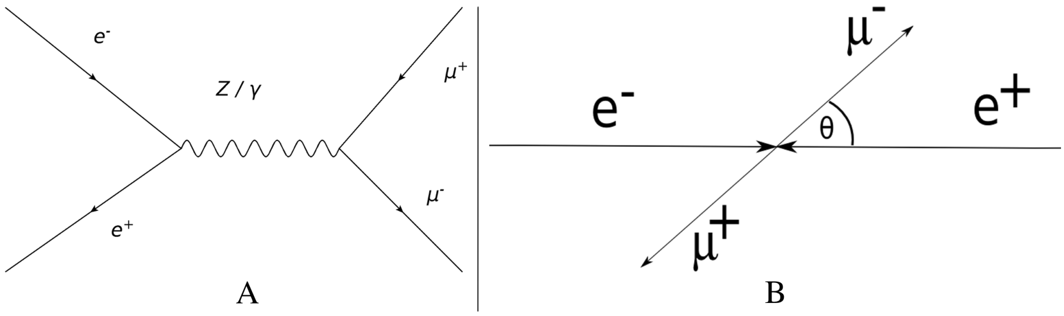

Figure 1: A. Feynman Diagram, B. Schematic Diagram of $e^+e^- \rightarrow \mu^+\mu^-$

Theoretical Formalism

Assuming the electron and positron are moving from opposite directions with the same amount of energy, i.e., in the centre-of-mass frame of the system [1], then the only particles produced, i.e., two muons, must come out back to back such that the momentum of the system is conserved. Shown in Fig. 1. B, angle θ is defined to be the angle between the positron direction and negative muon direction.

While photons produce muons symmetrically in angle θ, Z bosons do not [1]. However, it is the quantum superposition of a photon and a Z boson that decays into a pair of muons, and therefore does not necessarily produce muon pairs symmetrically in θ. The asymmetry ratio depends on the composition of the quantum superposition, which is effectively determined by the centre-of-mass energy.

For a specific collider running in a day, the angular distribution of produced muon pairs in $\cos\theta$ is given by

$$\frac{\mathrm{d}N_\mu}{\mathrm{d}\cos\theta} = \kappa[\sigma_S(\sqrt{s})(1+\cos^2\theta) + \sigma_A(\sqrt{s})\cos\theta], \tag{1}$$

[1]Fig. 1 is adapted from Computing Project B: Investigating asymmetry in particle collisions [1].

where N_μ is the total number of muon pairs, $\sqrt{s}$ is the centre-of-mass energy[2] and κ is a constant of the collider with unit[3] GeV^2. The σ_S and σ_A are defined as

$$\sigma_S(\sqrt{s}) = \frac{4}{3}\pi\left[\frac{1}{s} + \frac{sr_S + (s - M_Z^2)j_S}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right], \tag{2}$$

and

$$\sigma_A(\sqrt{s}) = \pi\left[\frac{sr_A + (s - M_Z^2)j_A}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right], \tag{3}$$

with unit GeV^{-2}, where M_Z and Γ_Z are the mass and decay width of Z bosons[4] [1].

The "r" parameters describe the peaks of σ_S and σ_A where the centre-of-mass energy is around M_Z. The "j" parameters describe the overall energy dependence of σ_S and σ_A[5]. The six parameters, M_Z, Γ_Z, r_S, r_A, j_S, j_A, can be used to construct a S-matrix that describes this entire scattering process [3].

The forward-backward asymmetry ratio is defined to be $\frac{\sigma_A}{\sigma_S}$ [1] as a function of centre-of-mass energy. It is directly related to the ratio of numbers of muon pairs distributed in $\cos\theta \geq 0$ (forward sphere) and $\cos\theta < 0$ (backward sphere). The numbers of muon pairs in the forward sphere and backward sphere can be respectively written as

$$N_\mu^f = \int_0^1 \frac{\mathrm{d}N_\mu}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta = \frac{4}{3}\sigma_S + \frac{1}{2}\sigma_A, \tag{4}$$

and

$$N_\mu^b = \int_{-1}^0 \frac{\mathrm{d}N_\mu}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta = \frac{4}{3}\sigma_S - \frac{1}{2}\sigma_A. \tag{5}$$

By applying Taylor expansion at the origin and assuming $\frac{3}{8}\frac{\sigma_A}{\sigma_S} \ll 1$, their ratio turns out to be

$$\frac{N_\mu^f}{N_\mu^b} = \frac{1 + \frac{3}{8}\frac{\sigma_A}{\sigma_S}}{1 - \frac{3}{8}\frac{\sigma_A}{\sigma_S}} \approx 1 + \frac{3}{4}\frac{\sigma_A}{\sigma_S}, \tag{6}$$

where $\frac{\sigma_A}{\sigma_S}$ can be identified to be the asymmetry ratio defined above.

[2] $\sqrt{s}$ is adopted to simplify the notation of square of energy used massively in relevant equations.

[3] Natural units are adopted, i.e., $c = \hbar = 1$.

[4] $M_Z = 91.2$ GeV, $\Gamma_Z = 2.5$ GeV [1].

[5] See Appendix A for more about S-matrix parameters.

Computational Methods

The experiments were simulated using Python 2.7 programming language with SciPy computing environment[6].

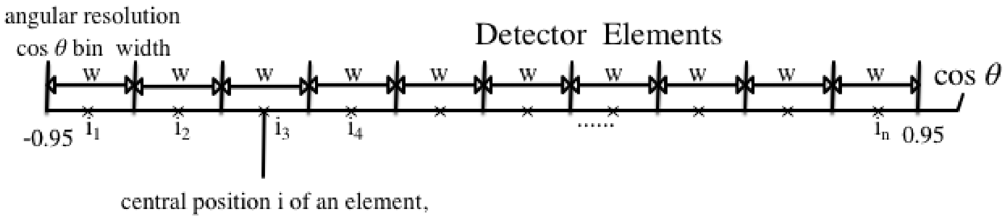

Figure 2: Schematic Diagram of Detector Elements

A collider was built with Eq. 1, where $\kappa = 1.0 \times 10^6\ GeV^2$, σ_S and σ_A defined as functions of centre-of-mass energy via Eq. 2 and 3. The collider was designed to operate at energies between 20 GeV and 140 GeV. Then, a detector was installed around the collision point, spanning a range of angles for $\cos\theta \in [-0.95, 0.95]$, left with two holes at the ends to enable electron and positron beams to enter. Shown in Fig. 2, the detector comprised a number of detector elements equally spaced in $\cos\theta$, with an angular resolution effectively equal to the width of the $\cos\theta$ bins.

The expected number of muon pairs detected by a single detector element is obviously the fraction of those produced by the collider and distributed in the range of that detector element. Thus, the expected number N_e^i for an element positioned at i, was calculated by integrating Eq. 1 with respect to $\cos\theta$ over the bin width, which is given by

$$N_e^i = \int_{i-w/2}^{i+w/2} \frac{\mathrm{d}N_\mu}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta = \kappa[\sigma_S(\cos\theta + \frac{1}{3}\cos^3\theta) + \frac{1}{2}\sigma_A\cos^2\theta]|_{i-w/2}^{i+w/2}, \quad (7)$$

where i is the central position of the detector element and w is the bin width.

However, the detector was actually counting the number of muon pairs, which introduced the random nature of a counting experiment. Hence, the expected number N_e^i was randomised by a Poisson distribution with N_e^i as the mean value. The effect of random noise, i.e., fake muon pairs detected, was also considered. For this specific detector, it was simulated by randomising 3 with another Poisson distribution to yield the number of fake muon pairs detected per day in each detector element. Then, the two randomised values were summed to give the

[6]NumPy, SciPy and PyLab libraries were used.

simulated number of muon pairs detected by a single detector element in a day, denoted by N_s^i. The process was repeated for all detector elements to give the angular distribution of N_s.

Since the angular distribution of muon pairs was modelled by Eq. 1, in order to extract the asymmetry ratio, the values of σ_S and σ_A were obtained by fitting Eq. 1 into the simulated distribution. To measure the energy dependence of asymmetry ratio, the process was repeated for energies between 20 GeV and 140 GeV. The resolution dependence of asymmetry ratio was also investigated for resolutions between 0.0095 and 0.475 such that the total number of detector elements is even for the detector to be symmetrical by itself.

The error on a counting experiment is the square root of number of counts [1]. So the error on N_s^i for each detector element was $\sqrt{N_s^i}$.

The errors on σ_S and σ_A are related to the error on N_s^i by the approximation to differential assuming the errors are small, which is given by

$$\delta N_s^i = \frac{\partial N_s^i}{\partial \sigma_S}\delta\sigma_S + \frac{\partial N_s^i}{\partial \sigma_A}\delta\sigma_A = \kappa[(1+\cos^2\theta)\delta\sigma_S + \cos\theta\delta\sigma_A], \tag{8}$$

where δN_s^i stands for the error on N_s^i, $\delta\sigma_S$ and $\delta\sigma_S$ are the errors on σ_S and σ_A. Then, the errors on σ_S and σ_A were obtained by fitting Eq. 8 into the data of $\sqrt{N_s^i}$ with corresponding $\cos\theta$. Subsequently, the error on the asymmetry ratio, $\frac{\sigma_A}{\sigma_S}$, was derived by combining the percentage errors of σ_S and σ_A.

To calculate the "r", "j" parameters, asymmetry ratio A was written as a function of $\sqrt{s}$:

$$A(\sqrt{s}) = \frac{\sigma_A(\sqrt{s})}{\sigma_S(\sqrt{s})}, \tag{9}$$

which is essentially Eq. 3 divided by Eq. 2. The "r", "j" parameters were derived by fitting Eq. 9 into the data of asymmetry ratio with corresponding centre-of-mass energies.

The errors on the "r", "j" parameters are related to the error on asymmetry ratio by again the approximation to differential provided the errors are small, which is given by

$$\delta A = \frac{\partial A}{\partial r_A}\delta r_A + \frac{\partial A}{\partial r_S}\delta r_S + \frac{\partial A}{\partial j_A}\delta j_A + \frac{\partial A}{\partial j_S}\delta j_S, \tag{10}$$

where δA is the error on asymmetry ratio, δr_A is the error on r_A, δr_S is the error on r_S and etc. Similar to the method of deriving errors on σ_S and σ_A, the errors of the "r", "j" parameters were obtained by fitting Eq. 10 into the data of errors on asymmetry ratio with corresponding centre-of-mass energies.

The errors originated from curve fit were taken into account for the calculation of the values of asymmetry ratio and the "r", "j" parameters, but neglected for the calculation of errors on them, which would introduce "error on error".

Results and Discussion

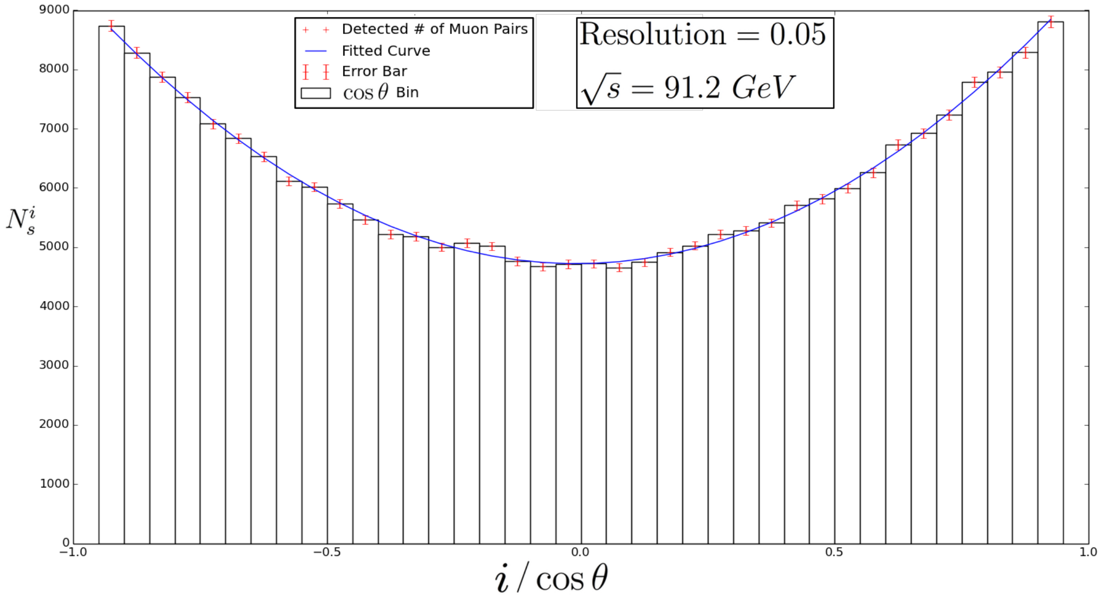

Figure 3: Simulated Angular Distribution of N_s^i in $\cos\theta$

The angular distribution of simulated number of muon pairs detected in a day without noise simulation is shown in Fig. 3[7]. A curve was fit to the distribution to extract the asymmetry ratio for different energies and resolutions.

The energy dependence and resolution dependence of asymmetry ratio were first studied by running the collider and detector in one day without simulation of random noise.

Shown in Fig. 4. A, by considering the approximation in Eq. 6, the collider was producing muon pairs almost symmetrically when energy was lower and at $\sqrt{s} = M_Z$, i.e., the number of muon pairs produced in forward sphere equals number in the backward when $\frac{\sigma_A}{\sigma_S} = 0$. The magnitude of asymmetry ratio was always

[7]Fig. 3–7 are modified from the figures generated by matplotlib in SciPy.

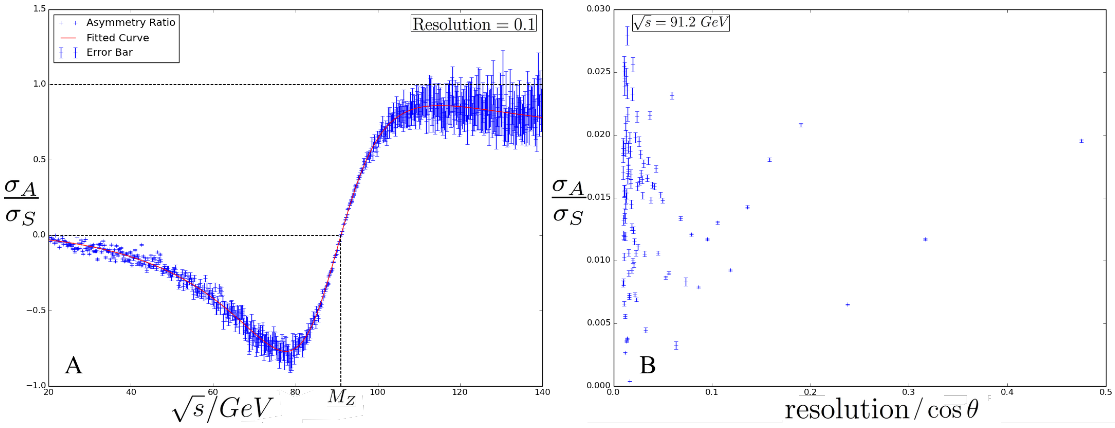

Figure 4: A. Energy Dependence, B. Resolution Dependence of Asymmetry Ratio

less than 1, which indicated that muon pairs produced in forward sphere ranged from significantly less than in the backward to almost as twice of in the backward. Moreover, the error on asymmetry ratio diminished around $\sqrt{s} = M_Z$. A curve was fitted to Fig. 4. A to calculate the "r", "j" parameters.

Asymmetry ratio was plotted against resolutions in Fig. 4. B, and there was no obvious trend on the resolution dependence of symmetry ratio.

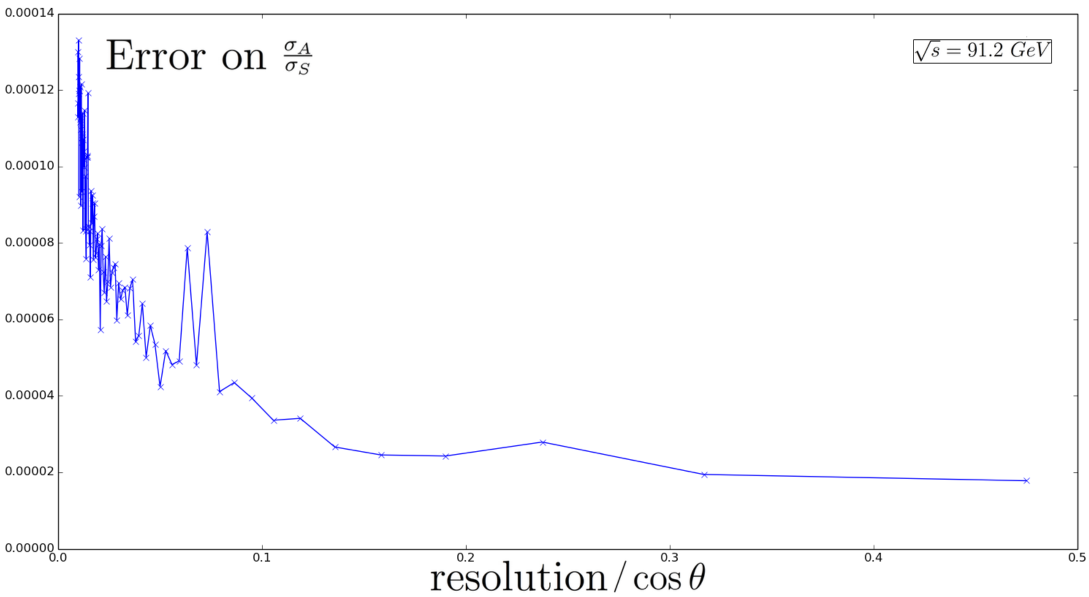

Figure 5: Resolution Dependence of Error on Asymmetry Ratio at 91.2 GeV

To choose an optimal resolution for the detector, the error on asymmetry ratio was analysed for different resolutions, shown in Fig. 5. Eventually the detector was built with resolution = 0.095, which had a total of 20 elements.

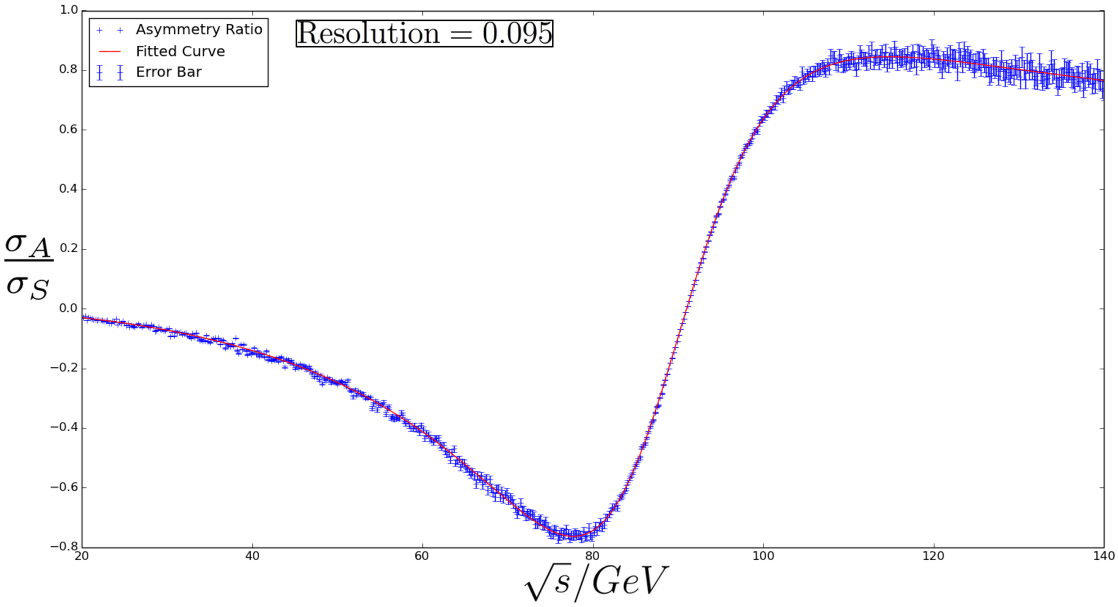

Figure 6: Energy Dependence of Asymmetry Ratio Simulated in 15 Days

The completed collider and detector were operated for 15 days with random noise simulation. The energy dependence of asymmetry ratio was shown in Fig. 6. The results obtained for the S-matrix parameters from the curve fit shown in red are

$$\begin{aligned} r_S &= 0.13829 \pm 0.00094, \\ j_S &= -0.029 \pm 0.025, \\ r_A &= 0.00296 \pm 0.00094, \\ j_A &= 0.7982 \pm 0.0049. \end{aligned}$$

Conclusion

The experiment was to investigate the asymmetry in particle collisions and calculate the S-matrix parameters for the process $e^+e^- \rightarrow \mu^+\mu^-$. The results are consistent with the experimental values obtained by ALEPH detector in 1997 [2]. The computing methods, error derivations in particular, could be significantly simplified if the approximation in Eq. 6 was assumed for this simulation project.

Bibliography

[1] David Colling, Carl Paterson, Computing Project B: Investigating asymmetry in particle collisions, Imperial College London, 30 Jan 2014.

[2] The AlEPH Collaboration (1997), Study of muon-pair production at centre-of-mass energies from 20 to 136 GeV with the ALEPH detector, PHYS. LETT. B, 399 (3-4) 329–341.

[3] A. Leike, T. Riemann, J. Rose, S-matrix approach to the Z line shape, PHYS. LETT. B273 (1991) 513.

Appendix A: S-matrix Parameterisation

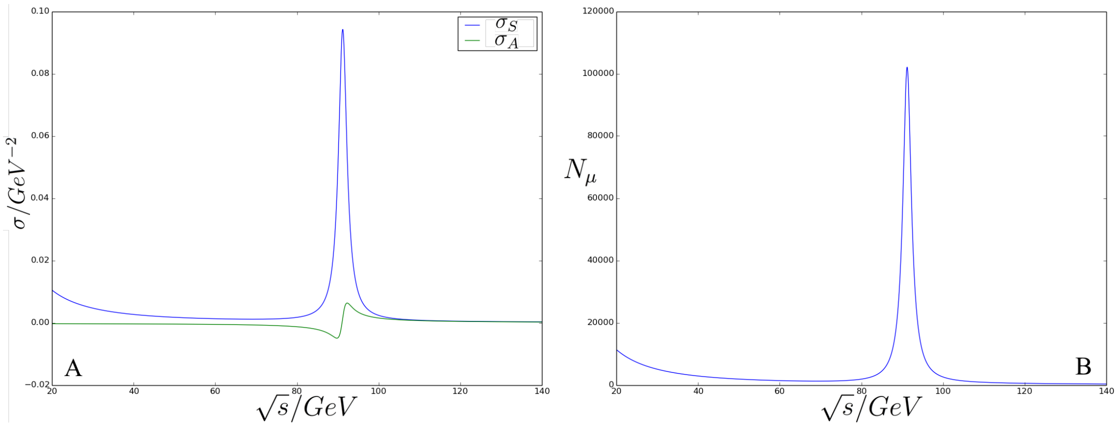

Figure 7: Energy Dependence of A. σ_S and σ_A, B. N_μ

The relations between σ_S, σ_A and centre-of-mass energy were plotted in Fig. 7. A. Z mass M_Z determines the position of the peaks, and the decay width Γ_Z affects the width of the peaks. While the r_S and r_A decide the height of the peaks of σ_S and σ_A respectively, the j_S and j_A describe the overall shapes and levels.

Shown in Fig. 7. B, the total number of muon pairs produced by the collider only depends on σ_S because the σ_A term cancels out when integrating Eq. 1 with respect to $\cos\theta$ from -1 to 1. As a consequence, the parameters that affect σ_S have the same effects on N_μ.

Appendix B: Programming Scripts