Research Statement

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Overview

My research interests lie broadly at the interface between theoretical physics and mathematics. In particular, I am captivated by the rich mathematical structures arising from supersymmetric quantum field theories. In the recent decades, the study of supersymmetric quantum field theories has proven to deliver striking insights into pure mathematics such as algebraic geometry, higher dimensional topology, non-commutative algebras, and representation theory.

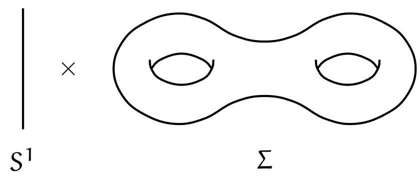
The focus of my research has been in those mathematical structures arising from 3d supersymmetric quantum field theories. The research in analogous theories in 2d has shown substantial contributions to the development of mathematics, such as homological mirror symmetry, Gromov-Witten theory, and quantum cohomology. These 3d supersymmetric theories possess analogous but distinct structures. Therefore it is expected that the study in 3d supersymmetric quantum field theories would also yield fruitful results in similar areas. My research aims to explore these mathematical structures arising in one dimension higher, e.g., quantum/quasi-map K-theory.

As someone coming from a physics background, this branch of research has been proven to be a very challenging but rewarding journey. I have had to quickly grasp a wide variety of abstract mathematical concepts, including characteristic classes, moduli spaces, equivariant cohomologies, K-theory, intersection theory, and mirror symmetry. But it has been immensely satisfying to see how these mathematical structures emerge and interact with physics.

During the past three years of my doctoral study, I have been researching on the algebro-geometric interpretations of exact solutions of a large class of 3d supersymmetric Chern-Simons gauge theories with matter [1], and their connections to quantum K-theory [2].

Geometry of Twisted Index

I obtained an algebro-geometric interpretation of 3d supersymmetric gauge theories via the technique of supersymmetric localisation [1]. This enabled us to introduce exact deformations and scaling limits that lead to different mathematical models of the same partition function. The focus was on the twisted index of 3d supersymmetric $\mathcal{N}=2$ gauge theories on $S^1 \times \Sigma$, where Σ is a closed Riemann surface of genus g [3–5]. The twist was performed using the unbroken R-symmetry, which preserved an $\mathcal{N}=(0,2)$ quantum mechanics on the S^1.



Using different classes of localisation schemes, the twisted index of the 3d $\mathcal{N}=2$ gauge theories localises to different types of integrals [8]. In Coulomb branch localisation scheme, the path integral localises onto configurations where the vectormultiplet scalar is non-zero and the gauge group is broken into a maximum torus [9–11]. This formulates the twisted index as Jeffrey-Kirwan contour integrals [3–5]. In Higgs branch localisation, the path integral localises onto configurations solving vortex equations. This interprets the twisted index as integrals of characteristic classes over the moduli spaces of vortices [12, 13].

I developed a Higgs branch localisation scheme, which lead to novel topological saddle points in addition to the vortex configurations. This was achieved by considering a different scaling limit in the path integral, with an exact deformation to the lagrangian depending on a 1d Fayet-Iliopoulos parameter [6]. It allowed us to unambiguously interpret the twisted index as the Witten index of the quantum mechanics on S^1, in each individual magnetic sector labelled by $\mathfrak{m}$, schematically given by [6, 7]

$$\mathcal{I} = \sum_{m \in \mathbb{Z}} q^m \int \hat{A}(\mathfrak{M}_m)\, \mathrm{ch}(\mathcal{E}_m)\,,$$

where $\mathfrak{M}_\mathfrak{m}$ denotes the moduli space parametrising saddle points of the localised path integral with magnetic flux $\mathfrak{m} \in \mathbb{Z}$, and $\mathcal{E}_\mathfrak{m}$ represents a complex of vector bundles arising from the massive fluctuations of chiral multiplets and Chern-Simons terms. For vortex saddles, the moduli space consists of symmetric products of the curve

$$\mathfrak{M}_m = \sum_i \mathrm{sym}^{d_i} \Sigma$$

for each possible non-vanishing chiral multiplet ϕ_i. For topological saddles, the moduli space is roughly a Picard variety parametrising holomorphic line bundles on Σ

$$\mathfrak{M}_m \sim \mathrm{Pic}^m(\Sigma) \simeq T^{2g}.$$

After a careful analysis of the index bundle and Chern-Simons terms, I was able to compute the characteristic classes using Groethendiek-Riemann-Roch theorem. This approach successfully reproduced the results obtained using the Jeffrey-Kirwan residue prescription under the conventional scheme of Coulomb branch localisation.

Chern-Simons Term and Quantum K-Theory

In addition to bridging these supersymmetric gauge theories to the field of enumerative geometry, these computations of the twisted index also opened up a new connection to quantum K-theory, which I have been exploring in the later half of my doctoral study.

Quantum K-theory is a K-theoretical extension to quantum cohomology developed in the 2000s [14–16]. Generally speaking, it studies the intersection theory of complex vector bundles over the spaces of holomorphic curves in Kähler manifolds. Many directions of quantum K-theory are under active research. For example, recent works [17–19] have developed deep connections to geometric representation theory and quantum integral systems. Furthermore, a striking correspondence between 3d gauge theories and quantum K-theory [22, 23] has also been under active research.

Quantum K-theory has an additional parameter called the "level" [20, 21], compared to quantum cohomology. We have proposed that this level can be interpreted as the Chern-Simons level in the class of supersymmetric gauge theories we studied before. The Higgs branch localisation developed in my previous research led to novel topological saddle points, in addition to the vortex saddle points. These topological saddle points exist when the effective Chern-Simons level in the asymptotic regions is non-zero. I have been able to demonstrate this phenomenon directly from the semi-classical vacuum equations, the Bethe ansatz equations, and the twisted indices. This phenomenon is a new physical interpretation to the "window" effect in quantum K-theory [24]. Crucially, this interpretation opens up the study on how to define quantum K-theory outside the window, by taking into account the topological saddles.

Research Direction

My passion remains at those mathematical structures emerging from quantum field theories. Although I have a plethora of ideas that I hope to build directly upon my doctoral research, I am more keen to learn about new topics and expand the scope of my research. The mathematical knowledge I accumulated during my doctorate has provided me a solid foundation to investigate other interesting theories.

One of my interests is mirror symmetry arising in 3d supersymmetric quantum field theories. Mirror symmetry in 2d has been a transformative tool for mathematics, particularly in the fields of algebraic geometry and symplectic geometry. It was used extensively in the study of quantum cohomology, stable maps, toric varieties, quantum differential equations, Floer theory, and A-infinity category. In comparison to 2d mirror symmetry, mirror symmetry in 3d has a distinct mathematical structure. It describes dualities between certain 3d supersymmetric gauge theories, where the Coulomb branch of one theory is mapped to the Higgs branch of the mirror theory. As these 3d supersymmetric gauge theories are deeply connected to geometry, progress on their dualities is likely to give new insights into geometry. I expect the research on 3d mirror symmetry to produce new mathematics. For example, it could be used to relate the quantum K-theories of mirror pairs, following my previous research.

Generalised global symmetry is another area I am eager to learn about. I would hope to investigate the implications of higher form symmetries on 3d supersymmetric gauge theories. In particular, in the presence of background fields for higher form symmetries, the modified twisted indices could lead to new mathematical structures. Furthermore, defects of higher form symmetries could lead to additional degeneracy on the Hilbert space.

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