Singlet Lens Optimisation and Rainbow Simulation

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March 23, 2015

Abstract

The project is to construct an optical ray tracing system to investigate the imaging performance of a singlet lens and simulate the formation of a rainbow. The curvatures of a biconvex singlet lens are optimised for specified refractive index, thickness and image distance by minimising spherical aberration. Compared with planoconvex lens of the same parameters, the optimised biconvex lens performs considerably better for a divergent incident beam but reduces to a planoconvex lens in case of a collimated beam. In addition, a rainbow is simulated by passing light beams of different wavelengths into an idealised spherical water drop with linear dispersion relation.

Introduction

An optical ray tracing system is a critical tool for optics design. [1] In this project, a basic ray tracer was coded to simulate refraction, reflection and dispersion of optical rays based on geometric optics. The optimisation of a singlet lens and simulation of rainbow formation were investigated using this basic ray tracing system.

Reflection is trivial from simple geometry, which simply reverts the normal component of a incident ray. For refraction, the transmitted unit vector can be derived from Snell's Law as

$$\mathbf{t} = \frac{\eta_1}{\eta_2}\mathbf{i} - (\frac{\eta_1}{\eta_2}\cos\theta_i + \sqrt{1 - |\mathbf{t}_{\parallel}|^2})\mathbf{n}, \tag{1}$$

where η_1, η_2 are the refractive indices, θ_i is the incident angle and $\mathbf{i}$, $\mathbf{t}$, $\mathbf{n}$ are the incident, refracted and normal unit vectors respectively. [2]

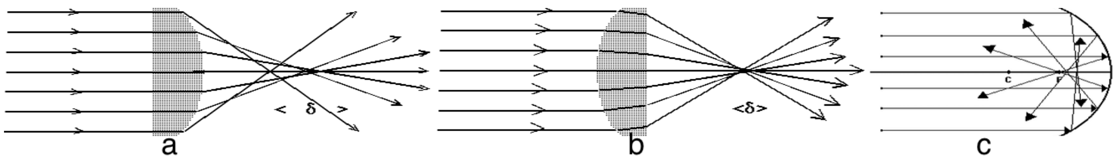

Figure 1: Spherical Aberration of Planoconvex Lens With Collimated Beam. [3]

Spherical aberration occurs when the refracted or reflected rays from a spherical optical component fails to converge to a single point as shown in Fig. 1. The rays further away from the optical axis of a spherical lens are refracted more. As a result, the produced images are blurred. It also occurs for a concave mirror. For imaging a collimated beam, the planoconvex lens of orientation a produces more aberration than orientation b.

Computational Methods

The ray tracing system was constructed with a sequential ray tracer, light beams and optical elements. The ray tracer was built with optical axis along z-axis to visualise light beams propagating through a variety of optical elements.

A single light beam was represented by a bundle of individual rays, which were a collection of vertices and directions at each vertex. The generated rays in a beam were spaced evenly to form concentric circles in the plane orthogonal to propagation. For example, a collimated beam, i.e., a pane wave, was was realised as a collection of rays pointing in the same direction and spaced evenly in concentric circles. A divergent beam, i.e., a spherical wave, was comprised of rays originating from a single point and spreading out in concentric circles.

The basic optical elements including refraction, reflection and an output plane were all based on a spherical interface with specific curvature, refractive index and optional circular aperture to reduce the effective radius, hence spherical aberration.

The compound optical elements were assembled using multiple basic elements. For example, a singlet lens was constructed with two refracting surfaces and a water drop for rainbow simulation consisted of a refractive front surface and a reflective back surface.

To make a best form singlet lens from a certain material, therefore refractive index, with a specified thickness and image distance, the lens were optimised by finding the curvatures of the two surfaces in a specified range, which minimise the RMS spot radius of a beam at the desired image distance. The RMS radius of the beam was calculated accounting for all the component rays. A planoconvex lens was considered as a constrained case of optimisation where the curvature of one surface was bounded to zero.

The curvature optimisation algorithm was implemented for both collimated beams and divergent beams. However, the intensity of a divergent beam would decay as the radius spread out, while a collimated beam would keep a constant intensity throughout propagation. To equalise the intensity of the beams incident on the lens, the object distance, i.e., the distance between the lens and source, was chosen such that the divergent beam spread just to the same radius as the collimated beam at the lens position. The desired object distance was calculated via $OD = R/\tan(\Theta/2)$, where R is the initial radius of the collimated beam, Θ is the beam divergence of the divergent beam. (Lab Book, p137) The performance of planoconvex lenses with the both orientations and biconvex lenses was measured and compared.

In order to simulate the formation of a rainbow, dispersion must be modelled for visible light beams. Multiple beams of different wavelengths between 380–780nm were directed onto an idealised spherical water drop with linear dispersion relation. The colours of different wavelengths were rendered by the ray tracer via an algorithm of converting wavelength to RGB values. The beams were refracted from the front surface into the water drop, then reflected from the back surface and eventually refracted out from the front surface to the output plane.

For a spherical water drop, total internal reflection on the back surface was not possible. The incident angle on the back surface approached to the critical angle when the incident beam on front surface moved away from the optical axis. It would only do equal to the critical angle when the incident beam was parallel to the spherical surface, where transmission would no longer go into the sphere. (Lab Book, p138)

Results and Discussion

The aberration of a collimated beam through a spherical biconvex lens was shown in Fig. 2. The first final spot shape was produced by spherical aberration with normal incident and the second one was due to coma aberration when the beam was incident at an angle with the optical axis. For spherical aberration, the outmost ring was deflected towards the centre more than the inner rings. In coma aberration, the circular symmetry was completely destroyed.

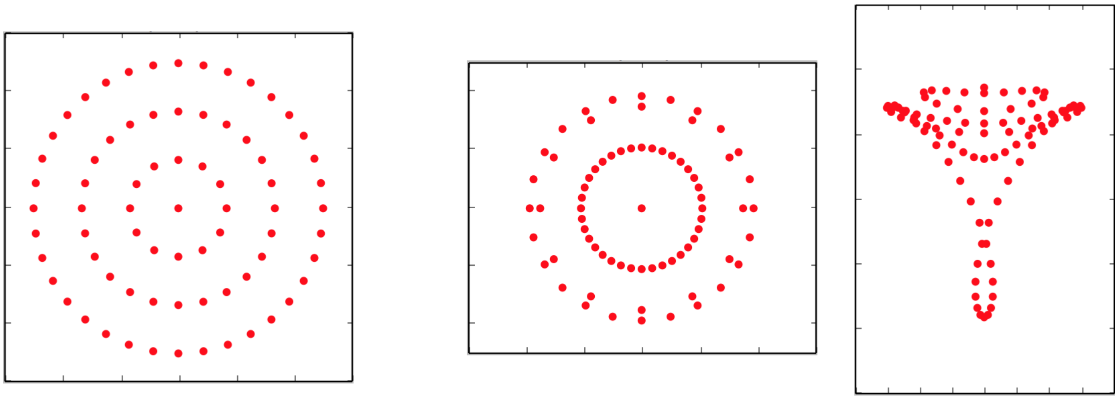

Figure 2: Spherical Aberration And Coma Aberration.

Shown in Fig. 3 was the comparison chart between planoconvex and biconvex lenses made of glass with refractive index $\eta = 1.15168$ for 588nm light rays. In the lens optimisation section, all the beams were generated to propagate along z-axis. The collimated beams were yielded with initial radius R = 5.00mm and the divergent beams were spread out from a single point source with beam divergence $\Theta = 10°$.

	Planoconvex	Planoconvex	Biconvex	Planoconvex	Planoconvex	Biconvex
Lens Thickness	5	5	5	5	5	5
Front Curvature	0	0.0192	0.0192	0	0.0524	0.0224
Back Curvature	-0.0198	0	0	-0.0513	0	-0.0317
Beam Divergence	0	0	0	10	10	10
Initial RMS Radius	2.85	2.85	2.85	0	0	0
Final RMS Radius	0.00336	0.000798	0.000798	0.097	0.138	0.0569
Object Distance	57.2	57.2	57.2	57.2	57.2	57.2
Image Distance	100	100	100	100	100	100

Note: Beam divergence was measured in degree.

Figure 3: Performance Comparison Between Planoconvex And Biconvex Lenses.

For imaging a collimated beam with zero beam divergence, the planoconvex lens with spherical surface facing the incident beam produced mush less aberration than the other orientation as indicated by its smaller RMS radius at image position, which agreed with the prediction in Fig. 1. However, the optimised biconvex lens turned out to be identical to the planoconvex lens, which meant the planoconvex lens was the best form lens in this situation.

In the case of imaging a divergent beam, the planoconvex lens with flat front surface performed better than its counterpart. The biconvex lens with optimised curvature had the best performance as expected, although the exact values of curvature must be calculated separately for different configurations.

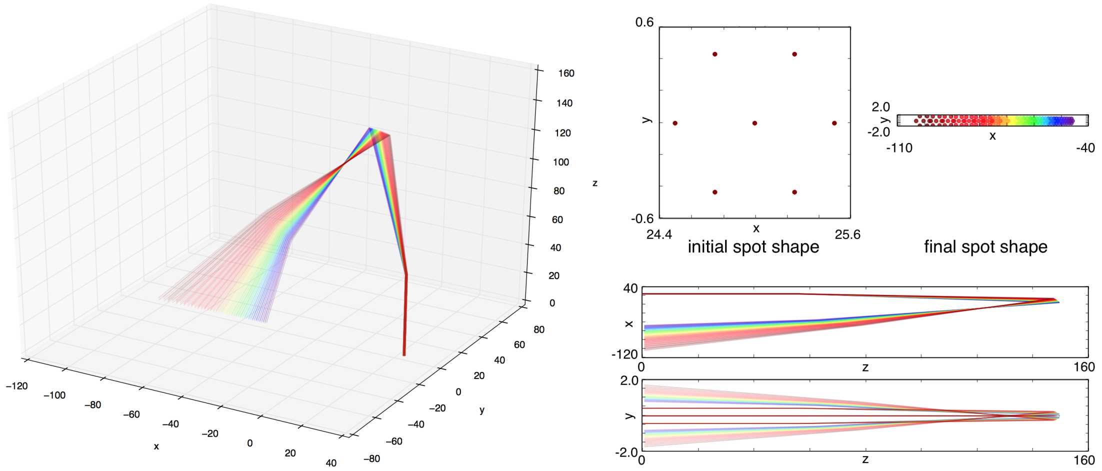

Figure 4: Rainbow Simulation With Spherical Water Drop.

Shown in Fig. 4, the rainbow simulation was realised by directing 20 light beams of wavelengths ranging from 380 to 760nm onto a spherical water drop. A model linear dispersion relation was assumed such that $\eta = -0.001 \times \lambda + 2$, where η is the refractive index and λ is the wavelength of the beam in nm. Since beams of longer wavelengths were refracted less than those of shorter wavelengths, each beam was deflected for different angle when coming out from the water drop. The rainbow pattern was therefore formed on the output plane as a direct visual representation of the proposed dispersion relation.

Conclusion

The experiment was to optimise the curvature of a singlet for converging both collimated beams and divergent beams, and simulate the formation of a rainbow. The best form lens for collimated beams was found to be the planoconvex lens with curved surface pointing towards the source. For divergent beams, the biconvex lens with optimised curvatures performed best in terms of minimal aberration. Furthermore, the rainbow was modelled using a spherical water drop of linear dispersion and visualised by the ray tracer. The ray tracing system implemented in this project could be easily extended to investigate more complicated optical systems.

Bibliography

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